

Calculators, Mobile Phones, Pagers and all other mobile communication equipment are not allowed.

1. a) Find $\frac{dy}{dx}$ if $y = [\sin(x^2 + 10)]^2$. (3 Points)

b) Let $x = \frac{x + xy + 2}{y^2}$. Find $\frac{dy}{dx}$ at $y = 1$. (3 Points)

2. a) Let $y = \frac{1}{r^2} - 2r$, and $r = 1 + \tan\left(\frac{2\pi}{t-1}\right)$. Find $\frac{dy}{dt}$ at $t = 3$. (3 Points)

b) Let $y = \frac{\sqrt{x}}{x-2}$. Approximate the change in y , if x changes from 4 to 3.9. (3 Points)

3. Find two real numbers x and y of minimum product such that $x + y^2 = 12$. (3 Points)

4. a) State The Mean Value Theorem.

b) Use The Mean Value Theorem to show that if $0 < a < b$, then

$$\sqrt{b} - \sqrt{a} < \frac{b-a}{2\sqrt{a}}$$

(4 Points)

5. Let $f(x) = \frac{x^2}{x^2-1}$, $f'(x) = \frac{-2x}{(x^2-1)^2}$ and $f''(x) = \frac{6x^2+2}{(x^2-1)^3}$.

a) Find the vertical and horizontal asymptotes (if any).

b) Find the intervals on which f is increasing and the intervals on which f is decreasing. Find the local extrema (if any).

c) Find the intervals on which the graph of f is concave upward and the intervals on which the graph of f is concave downward. Find the point of inflection (if any).

d) Discuss the symmetries of the graph of f .

e) Sketch the graph of f .

(6 Points)